Exploiting Redundant Measurements for Time Scale Generation in a Swarm of Nanosatellites

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***Abstract*—The computation of a common reference time for a swarm of nanosatellites is restricted by the quality and availability of the timing measurements made with inter-satellite links. The presence of anomalies or absence of communication links is demonstrated to harm the stability of the time scale. The Least Squares (LS) estimator is introduced as a method of preprocessing measurement noise by using all available clock comparisons in the swarm. This estimator also provides filtered measurements when inter-satellite links are missing as long as each satellite maintains at least one link with another. Anomaly detection and removing corrupted satellite links are shown to be compatible with the LS estimator to mitigate the impact of anomalous measurements. When a satellite becomes completely isolated for some period of time, a correction at the beginning and the end of the isolation period are both detailed. The correction is simple and just requires resetting the weights of missing clocks and clocks being reintroduced. Continuity is shown to be maintained when a large portion of clocks are removed and later reintroduced at the same time.**

***Index Terms*—Time scales, satellite constellation, clock comparisons, missing data**

I. Introduction

The issues of measurement noise and missing data are prominent when considering large-scale constellations of nanosatellites. Autonomous synchronization in satellite swarms is achieved by generating a time scale using only the onboard clocks. Specifically, time scales for missions performing interferometric radio astronomy in space [1] are susceptible to timing instabilities due to corrupted or unavailable measurements. In these applications, two-way transfers measure the clock biases between satellites with non-negligible and non-uniform measurement noise. Additionally, abrupt changes can occur in the noise affecting certain links, resulting in anomalous measurements or missing measurements.

A robust time scale algorithm aims to assign low weights to anomalies, which requires some method of detecting anomalies. Since anomaly detection methods are well represented in the literature [2, 3, 4], a capacity to detect the anomalies is assumed in this paper. The contribution of this work is to process corrupted measurements when there is a reduced amount of total available measurements as well as to maintain stability in the time scale when there are missing measurements.

This research has been conducted as part of a PhD thesis cofunded by the research laboratory TéSA and the Centre National d'Études Spatiales (CNES).

The mitigation of measurement noise and anomalies is based on the Least Squares (LS) estimator. The estimator is introduced in Section III, where it reduces the impact of the measurement noise and reconstructs specific missing measurements using additional information on the satellite time differences. The advantage of detecting and removing anomalous inter-satellite links is also analyzed by showing the LS estimator's ability to reproduce the removed measurements with reduced error compared to direct measurements.

A modification of clock weights is demonstrated to ensure time scale continuity when clocks are removed from the ensemble in Section IV. Isolation can be caused by communication limits or the removal of associated measurements due to an internal clock anomaly. By applying an appropriate correction when a clock goes missing, the time scale remains stable for the remaining clocks. A similar correction is required to reintroduce isolated clocks when they become available again. Section V describes the requirements to ensure a smooth reintroduction and how the AT1 time scale naturally provides a solution after resetting the weights of reintroduced clocks.

II. Measurement model

Time differences between onboard clocks are measured to construct an autonomous time scale in a swarm of nanosatellites. A two-way transfer method allows simultaneous measurement of the geometric range $R_{A,B}$ and time difference $x_{A,B}$ between two satellites A and B. The resulting measurements retain some noise that may be augmented by anomalies occurring on certain communication links.

The measurements with and without measurement noise can be defined in two vectors:

$$\mathbf{z}(t) = [z_{1,2}(t), \cdots, z_{1,N}(t), z_{2,3}(t), \cdots, z_{(N-1),N}(t)]^T, \quad (1)$$

$$\mathbf{x}(t) = [x_{1,2}(t), \cdots, x_{1,N}(t)]^T, \quad (2)$$

where the noisy measurements $z_{i,j}(t) = x_{i,j}(t) + n_{i,j}(t)$ include all possible unique links between satellites and their associated noise. The noiseless measurements in $\mathbf{x}(t)$ are simply the $N-1$ unique and non-redundant measurements required to compute the Basic Time Scale Equation (BTSE).

To understand how measurement noise affects the computation of a time scale, the BTSE is presented as a sum between the standard BTSE in [5, 6] and the remaining noise:

$$x_{i,E}(t) = \sum_{j=1}^{N} w_j(t-\tau)\left[\hat{x}_{j,E}(t) - x_{j,i}(t)\right] - \sum_{j=1}^{N} w_j(t-\tau) n_{j,i}(t), \tag{3}$$

where the predictions $\hat{x}_{j,E}(t)$ are made using $x_{i,E}(t-\tau)$ and a frequency estimate $y_{i,E}(t-\tau)$, where τ is the interval since the previous measurements were available. The BTSE estimates the time difference between the clock in satellite i and the ensemble reference time. Generally, the reference should be identical regardless of which satellite is responsible for taking measurements. However, the remaining noise terms $n_{j,i}(t)$ depend on the quality of the links between satellite i and the rest of the swarm. Although the noise is averaged over the N satellites, the magnitude can be amplified by anomalies on certain links and cause jumps in the time scale for only certain satellites.

Appropriately reducing the weights corresponding to anomalous clocks and corrupted measurement links allows the time scale to be robust to problems faced in the swarm. However, this method requires anomaly detection and quantification to adjust weights correctly. This work instead considers the case of missing measurements as a generalization for dealing with all types of anomalies. That is, contaminated measurements can be removed from the data set if they are detected, becoming missing measurements. As a result, the proposed methodology to deal with missing communication links can be extended to removing measurements with elevated noise.

III. Noise and anomaly mitigation with Missing measurements

Assuming that neighboring satellites can communicate all unique clock comparisons between them, a total of $N(N-1)/2$ unique observations are available in a swarm of N satellites. With negligible noise, $N-1$ unique and independent measurements are required to generate a time scale using the BTSE. As shown in (3), noise in the measurements introduces uncertainty in the realization of a time scale. Since each time difference measurement can be written as a linear combination of the other measurements, the noise can be reduced with an LS estimator for the $N-1$ required measurements. That is, the set of all possible clock comparisons $\mathbf{z}(t)$ can be written as a set of linear equations of the required measurements $\mathbf{x}(t)$. An example is written for the case of $N=4$ satellites but can be easily expanded to greater numbers of satellites

$$\mathbf{z}(t) = \begin{bmatrix} z_{1,2}(t) \\ z_{1,3}(t) \\ z_{1,4}(t) \\ z_{2,3}(t) \\ z_{2,4}(t) \\ z_{3,4}(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_{1,2}(t) \\ x_{1,3}(t) \\ x_{1,4}(t) \end{bmatrix} + \begin{bmatrix} n_{1,2}(t) \\ n_{1,3}(t) \\ n_{1,4}(t) \\ n_{2,3}(t) \\ n_{2,4}(t) \\ n_{3,4}(t) \end{bmatrix}, \tag{4}$$

or otherwise,

$$\mathbf{z}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{n}(t). \tag{5}$$

The matrix $\mathbf{A}(t)$ depends on time because its number of rows depends on the number of available measurements. For example, the link between satellites 1 and 2 may not be physically possible to obtain or it could be neglected due to a detected anomaly. As a result, the first row would not appear in $\mathbf{A}(t)$. The LS estimator is defined as:

$$\hat{\mathbf{x}}_{LS}(t) = \min_{\mathbf{x}(t)} \left\{ \|\mathbf{z}(t) - \mathbf{A}(t)\mathbf{x}(t)\|^2 \right\}, \tag{6}$$

$$\hat{\mathbf{x}}_{LS}(t) = \left(\mathbf{A}(t)^T \mathbf{A}(t)\right)^{-1} \mathbf{A}(t)^T \mathbf{z}(t) \tag{7}$$

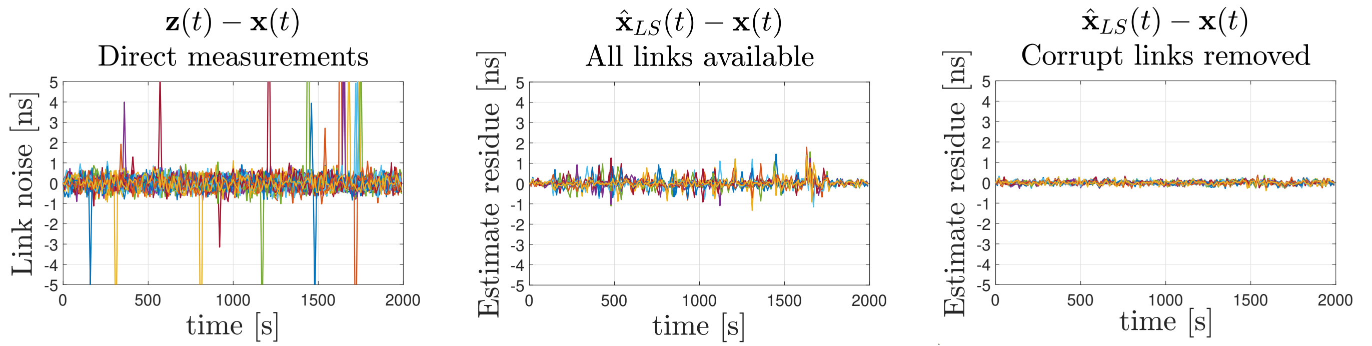
The number of measurements in $\mathbf{z}(t)$ will reduce for each unavailable or removed measurement, hence, the number of rows in matrix $\mathbf{A}(t)$ will also decrease. While the rank of this matrix remains equal to $N-1$, the LS estimator can still estimate the required measurements contained in $\mathbf{x}$ with residual error proportional to the number of available links.

To visualize the noise reduction achievable by the LS estimator, a situation with $N = 25$ satellites is simulated to represent a swarm of nanosatellites. All inter-satellite clock comparisons are simulated with Gaussian zero-mean additive measurement noise with a standard deviation of 0.3 ns. Anomalies of 10 ns are introduced at randomly chosen times on each unique link, some links may have anomalies at the same instant but each link only has one anomaly. Figure 1a indicates the intensity of the measurement noise on the $N-1$ required measurements made with respect to satellite 1. These measurements are sufficient to be substituted directly into (3) but will result in an unstable reference.

The LS estimator provides estimates with a residue lower than the noise shown in Figure 1b. The anomalous measurements are not removed so that the LS estimator can be assessed in the case of non-detection. The anomalies are seen to be reduced because there is a sufficient number of additional measurements available simultaneously without anomalies.

With one out of 300 possible measurements affected by an anomaly at a single time instant, the spike due to that anomaly is noticeably filtered. However, when one of the required measurements, e.g., $x_{1,2}(t)$ has no anomaly, an anomaly occurring on one of the redundant measurements, e.g., $x_{3,4}(t)$ can cause the estimate to have a significant deviation compared to the direct measurement at that time instant. Depending on the magnitude of measurement noise and the magnitude of the anomaly, either the direct measurement or the LS estimator could be preferred.

The LS estimator still functions with certain measurements removed. Therefore, measurements classified as anomalies can be removed and the LS estimator can use the additional measurements to estimate those removed measurements. Figure 1c illustrates the benefit of removing the anomalous measurements (assuming they can be perfectly detected) by showing that the error is reduced in comparison to the direct measurements seen in Figure 1a.



(a) Direct measurement noise with a single anomaly occurring on each link at a random point in time.

(b) Remaining error on the LS estimator when using all possible unique clock comparisons.

(c) Remaining error on the LS estimator when removing all measurements marked as anomalies to estimate.

Fig. 1: Least squares estimator noise reduction for the 24 inter-satellite links measuring the time differences $x_{1,2}, \cdots, x_{1,25}$ by using redundant measurements in the satellite swarm.

If an anomaly affects the clock states, the number of measurements removed results in a reduction of the rank of $\mathbf{A}(t)$. In this case, the weights and the BTSE should be modified at the instant the measurements are removed. This is similar to the case of dealing with completely isolated clocks, which is explained in the next section.

IV. Removal of Missing Clocks

A swarm has access to N satellites in the best-case scenario. Considering N_m satellites can lose connection from the swarm at time t_m, how do we maintain continuity in the time scale when only using measurements from the remaining $N_a = N - N_m$ clocks? The change in the time scale due to having a reduced number of clocks is defined as:

$$C_{N_m}(t_m) = x_{i,E}(t_m)|_N - x_{i,E}(t_m)|_{N_a}, \tag{8}$$

where the time scale computed with the remaining N_a clocks is

$$x_{i,E}(t_m)|_{N_a} = \sum_{j=1}^{N_a} u_j(t_m - \tau) r_{j,i}(t_m), \tag{9}$$

and the time scale if all clocks were still available, is

$$x_{i,E}(t_m)|_N = \sum_{j=1}^{N_a} w_j(t_m - \tau) r_{j,i}(t_m) \tag{10}$$

$$+ \sum_{j=N_a+1}^{N} w_j(t_m - \tau) r_{j,i}(t_m), \tag{11}$$

which is split into two sums for the remaining and missing clocks. Neglecting the fact that $r_{j,i}(t_m)$ is not available for the missing clocks, we can compute the change in the time scale

$$C_{N_m}(t_m) = \sum_{j=1}^{N_a} \left(w_j(t_m - \tau) - u_j(t_m - \tau)\right) r_{j,i}(t_m) \tag{12}$$

$$+ \sum_{j=N_a+1}^{N} w_j(t_m - \tau) r_{j,i}(t_m). \tag{13}$$

The weights that were computed at the previous time instant are used to generate the AT1 time scale, with weights calculated for the full system denoted as $w_j(t - \tau)$, and weights computed for the reduced system denoted as $u_j(t - \tau)$. Since the weights of the missing clocks were computed using the information from the previous time epoch, they do not consider that those clocks are missing at $t = t_m$. Since it is simple to detect which clocks are missing at $t = t_m$, the weights of those clocks can be set to zero. If the weights of the missing clocks are set to zero then the remaining weights must be renormalized to maintain the uniqueness of the time scale for each clock. This means that the weights should be modified such that

$$w_j(t_m - \tau) = w_j(t_m - \tau) \Bigg/ \sum_{i=1}^{N_a} w_i(t_m - \tau)\,, \quad j \le N_a, \tag{14}$$

$$w_j(t_m - \tau) = 0, \qquad j > N_a. \tag{15}$$

If the above values are substituted into (13) then the expression for the change in the time scale due to losing N_m clocks becomes

$$C_{N_m}(t_m) = \sum_{j=1}^{N_a} \left(w_j(t_m - \tau) - u_j(t_m - \tau)\right) r_{j,i}(t_m). \tag{16}$$

This can be set to zero, ensuring phase continuity of the time scale by setting $u_j(t_m - \tau) = w_j(t_m - \tau)$ for the weights used to compute the reduced time scale at $t = t_m$. This proves why resetting the weights of missing clocks to zero allows continuity in the time scale when the number of available clocks changes. The above derivation can similarly justify continuity for an increasing number of clocks in the ensemble. However, if the weights always stay at zero for new (or returning) clocks then the time scale will never benefit from the increased number of clocks available.

V. Reintroduction of Missing clocks

When inter-satellite links are reintroduced into the swarm at $t = t_r$, the time scale must again ensure continuity to avoid instantaneous jumps by ensuring the returning clocks have zero weights when computing $x_{i,E}(t_r)$. Since the weights of the missing clocks were already reset in the past, $w_j(t_r - \tau) = 0$ is guaranteed for returning clocks j and $x_{j,E}(t_r)$ is computed using the non-returning clocks. At the time instant after the clocks have returned ($t = t_r + \tau$), the new clock measurements were still effectively "missing" at the previous epoch but there is some information provided by $x_{j,E}(t_r)$.

To profit from the new measurements, the weights must eventually be non-zero, but they must not change rapidly because that will cause a jump in the time scale. Hence, a gradual reintroduction of the weights for the new clocks is used after the first reset at $t = t_r$. The AT1 time scale algorithm is compatible with this requirement thanks to an exponential filter that gradually increases weights over time according to an optimally chosen time constant. This time constant defines the amount of time before the reintroduced clocks are capable of contributing to the time scale again.

Figure 2 illustrates the resulting time scale with and without the appropriate weights being reset at the required times. AT1R is used to refer to the AT1 time scale using reset. The time scale is computed with 10 out of 50 total simulated clocks being removed at once. All of the isolated clocks are reintroduced simultaneously after 50 minutes. This is expected in the application of a swarm of satellites. The results prove that continuity is still maintained with something as simple as forcing the weights to zero at the appropriate times.

Although continuous, the overall stability of the time scale is still reduced compared to the case with no isolated clocks (see the dashed blue line with crosses in Figure 2). This is unavoidable because the optimal reduction in Allan deviation (ADEV) for a time scale is proportional to the number of clocks. The ADEV is estimated using simulated samples over 20000 seconds with a data rate of 10 seconds. This means that the outage period of 3000 seconds for the missing clocks has a marginal effect on the overall ADEV. If the ADEV were instead estimated over only the duration of isolation, the time scale stability would be degraded to the achievable level for the available clocks.

VI. Conclusion and Future work

Given enough linearly independent measurements, the proposed LS estimator can estimate the necessary $N - 1$ measurements to compute each satellite's time difference from the common time scale. With sufficient inter-satellite links, the effects of noise and anomalies in direct measurements are reduced in the LS estimates. This is still true even if certain inter-satellite links are broken because the remaining links provide redundancy. Assuming perfect detection of anomalies and removing the affected measurements, the resulting jumps can be eliminated and the removed measurements can still be estimated with the LS estimator. Specific anomaly detection algorithms should be investigated in future work to test the corresponding benefits alongside the LS estimator and redundant measurements. The method of resetting weights maintains continuity in the time scale when clocks become isolated due to broken communication links or removed due to anomalies. That being said, the reintroduction of missing clocks requires a gradual increase from the reset weights like that offered in the exponential filter of the AT1 algorithm.

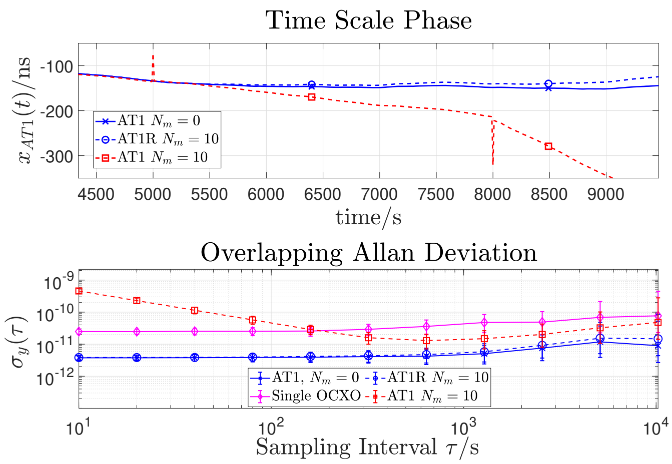


Fig. 2: Discontinuities in the time scale phase (top) appear when weights are not reset at $t_m = 5000$ s or $t_r = 8000$ s. As a result, the frequency stability is reduced, evidenced by a higher Allan deviation for the resulting time scale without reset (bottom).

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