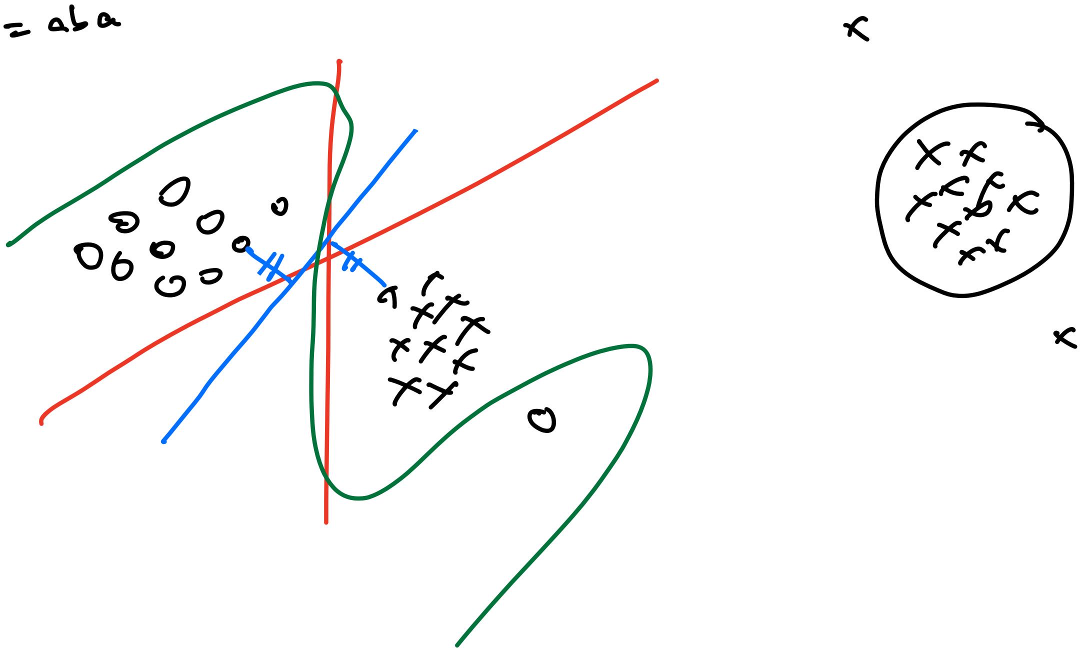
$s_1 = abc$ $\quad$ $d(s_1, s_2) = 3$
$s_2 = bca$

$s_1 = abc$ $\quad$ $d(s_1, s_2) = 1$
$s_2 = aba$



$\boxed{\text{Cours du } 19/02/2026}$

$$\frac{\partial}{\partial \beta_j} \ln h_\beta(x^{(i)}) \; ? \qquad \frac{\partial}{\partial \beta_j} \ln\left[1 - h_\beta(x^{(i)})\right] \; ?$$

$$h_\beta(x^{(i)}) = \frac{1}{1 + e^{-\beta^T x^{(i)}}} = \frac{1}{1 + \exp\big(\underbrace{-\beta_0 x_0^{(i)} - \beta_2 x_2^{(i)} \cdots - \beta_p x_p^{(i)}}_{-\beta^T x^{(i)}}\big)}$$

$$= \sigma(\beta^T x^{(i)})$$

$$\sigma(z) = \frac{1}{1 + e^{-z}} \qquad 1 - \sigma(z) = 1 - \frac{1}{1 + e^{-z}} = \frac{e^{-z}}{1 + e^{-z}}$$

$$\sigma'(z) = \frac{e^{-z}}{(1 + e^{-z})^2} = \sigma(z)\left[1 - \sigma(z)\right]$$

$$\boxed{\sigma'(z) = \sigma(z)\left[1 - \sigma(z)\right]}$$

$$\frac{\partial h_\beta(x^{(i)})}{\partial \beta_j} = \sigma(\beta^T x^{(i)})\left[1 - \sigma(\beta^T x^{(i)})\right] \underbrace{\frac{\partial \beta^T x^{(i)}}{\partial \beta_j}}_{x_j^{(i)}}$$

$$\boxed{\frac{\partial h_\beta(x^{(i)})}{\partial \beta_j} = \sigma(\beta^T x^{(i)})(1-\sigma(\beta^T x^{(i)}))\, x_j^{i}}$$

$\rightarrow h_\beta(x^{(i)})$ $\rightarrow 1 - h_\beta(x^{(i)})$

On en conclut :

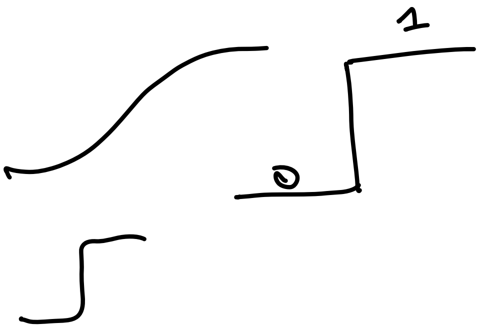
$$\frac{\partial \ln h_\beta(x^{(i)})}{\partial \beta_j} = \frac{1}{h_\beta(x^{(i)})} \frac{\partial h_\beta(x^{i})}{\partial \beta_j}$$

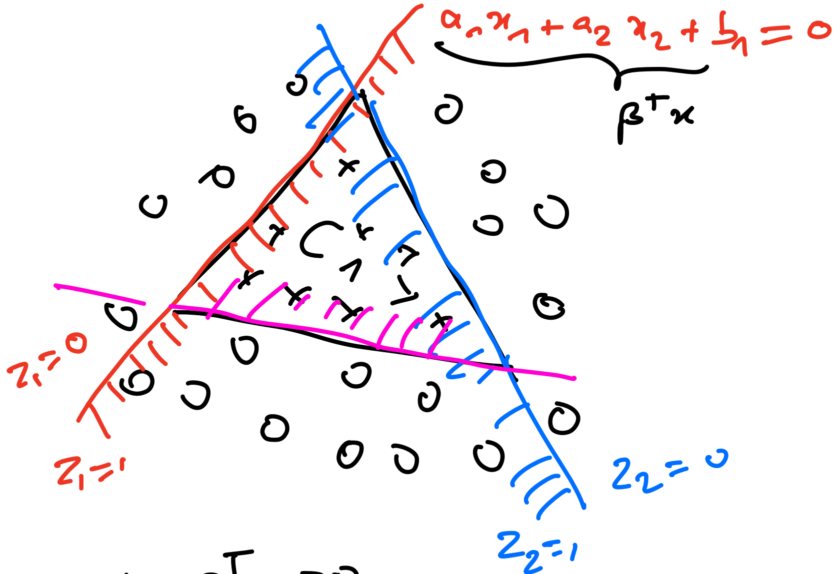
donc $$\boxed{\frac{\partial \ln h_\beta(x^{(i)})}{\partial \beta_j} = [1 - h_\beta(x^{(i)})]\, x_j^{(i)}}$$

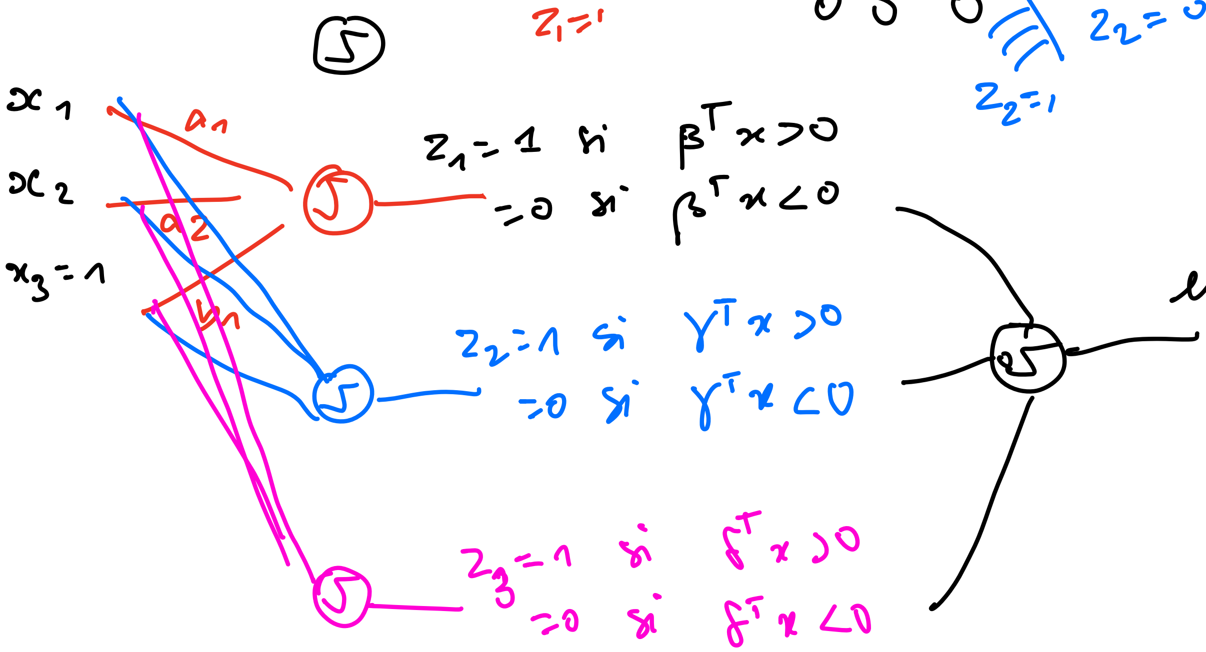
$$\frac{\partial \ln(1 - h_\beta(x^{(i)}))}{\partial \beta_j} = \frac{-1}{1 - h_\beta(x^{(i)})} \frac{\partial h_\beta(x^{(i)})}{\partial \beta_j}$$

$$\boxed{\frac{\partial \ln(1 - h_\beta(x^{(i)}))}{\partial \beta_j} = -h_\beta(x^{(i)})\, x_j^{(i)}}$$

Cours du 26/02/2026







$$u = \sigma(w_1 z_1 + w_2 z_2 + w_3 z_3 + b_3) = \sigma(z_1 + z_2 + z_3 + b_3)$$

$$w_1 = w_2 = w_3 = 1$$

si $(x_1, x_2, x_3) \in \Delta$, alors $z_1 = z_2 = z_3 = 1$

donc $u = \sigma(3 + b_3)$

$= 1 \iff b_3 + 3 > 0$

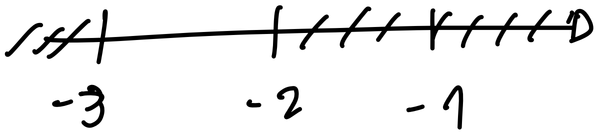
$\boxed{b_3 > -3}$

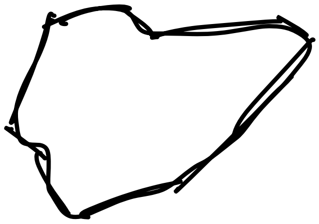
Si $(x_1, x_2, x_3) \notin \Delta$, alors $\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} \in \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \ldots \right\}$

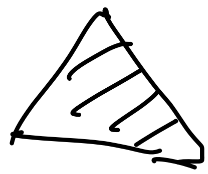
si $\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \Rightarrow u = \sigma(2 + b_3)$

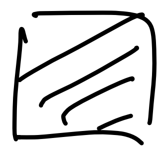
on veut $u = 0 \Leftrightarrow 2 + b_3 < 0 \Leftrightarrow \boxed{b_3 < -2}$

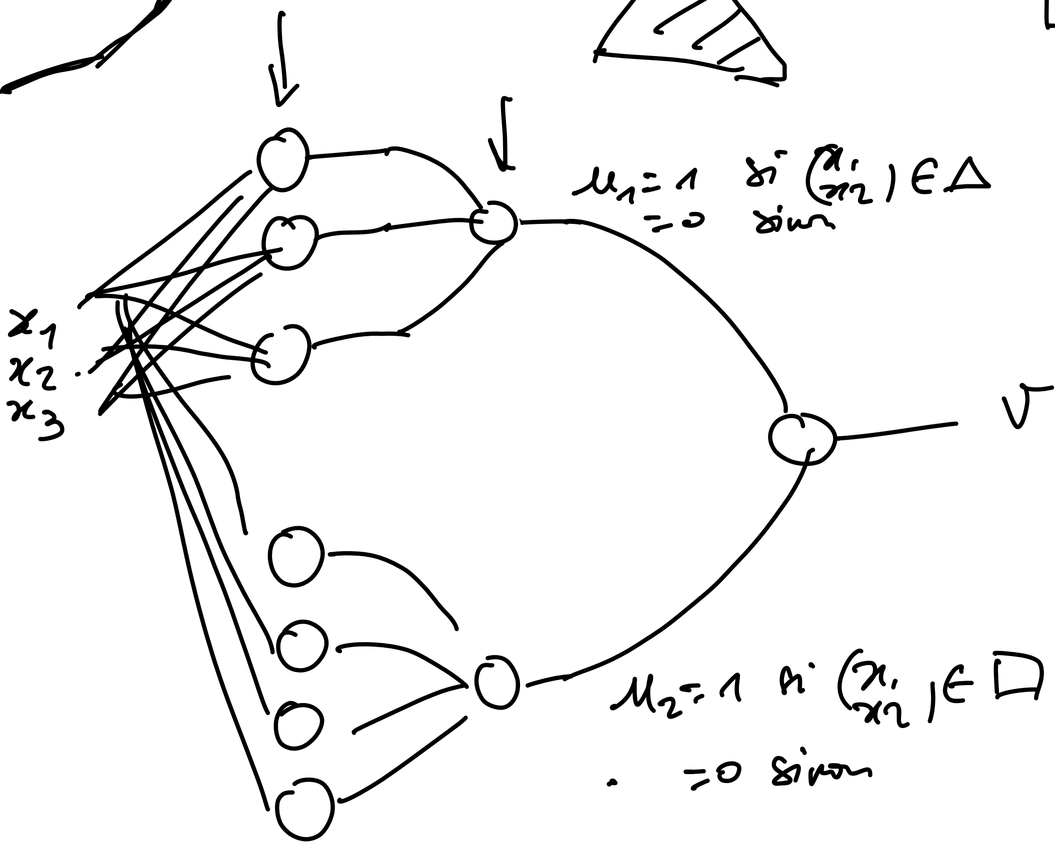
si $\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Rightarrow u = \sigma(1 + b_3) = 0 \Leftrightarrow \boxed{b_3 < -1}$











$u_1 = 1$ si $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \Delta$
$= 0$ sinon

$u_2 = 1$ si $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \square$
$= 0$ sinon

$v = f(u_1 + u_2 + c)$

si $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \square$ $\quad u_1 = 0$, $u_2 = 1$

$v = f(1 + c) = 1$

$\Leftrightarrow 1 + c > 0$

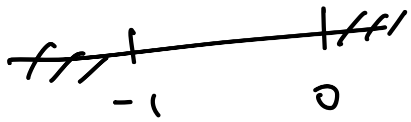
$\boxed{c > -1}$

si $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \Delta$ $\quad u_1 = 1$, $u_2 = 0$

$v = f(1 + c) = 1 \Leftrightarrow \boxed{c > -1}$

si $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \notin \Delta \Rightarrow \begin{matrix} \mu_1 = 0 \\ \mu_2 = 0 \end{matrix} \Rightarrow \sigma = f(c) = 0 \Leftrightarrow \boxed{c < 0}$

$\notin \Delta$



Rétropropagation du gradient (backpropagation)

$$\hat{y} = \sigma(w_1 h_1 + w_2 h_2 + b_3)$$

$$\text{Loss} = \frac{1}{n} \sum_{i=1}^{n} \left(y^{(i)} - \hat{y}^{(i)} \right)^2$$

$$= \frac{1}{n} \sum_{i=1}^{n} \Big(\underset{\in\{0,1\}}{y^{(i)}} - \underbrace{\sigma\left(w_1 h_1^{(i)} + w_2 h_2^{(i)} + b_3 \right)}_{h_\beta(x^{(i)})} \Big)^2$$

Règle du gradient

$$w^{\{k+1\}} = w^{\{k\}} - \alpha \left. \frac{\partial \text{Loss}}{\partial w} \right|_{w = w^{\{k\}}}$$

$$w_1^{\{k+1\}} = w_1^{\{k\}} + \alpha \frac{1}{n} \sum_{i=1}^{n} 2 \left(y^{(i)} - h_\beta(x^{(i)}) \right) \underbrace{h_\beta(x^{(i)}) \left(1 - h_\beta(x^{(i)}) \right)}_{\sigma'}$$

$\times h_1^{(i)}$

$$w_2^{\{k+1\}} = w_2^{\{k\}} + \frac{\alpha}{n} \sum_{i=1}^{n} 2 \left(y^{(i)} - h_\beta(x^{(i)}) \right) h_\beta(x^{(i)}) \left(1 - h_\beta(x^{(i)}) \right)$$

$\times h_2^{(i)}$

$$b_3^{\{k+1\}} = b_3^{\{k\}} + \frac{\alpha}{n} \sum_{i=1}^{n} 2 (\cdots) \times 1$$

Mise à jour des poids de la 1ère couche

$$w_{11}^{\{k+1\}} = w_{11}^{\{k\}} - \alpha \left.\frac{\partial \text{loss}}{\partial w_{11}}\right|_{w_{11}=w_{11}^{\{k\}}}$$

$$= w_{11}^{\{k\}} - \alpha \frac{1}{n}\sum_{i=1}^{n} 2\left(y^{(i)} - h_\beta(x^{(i)})\right)$$

$$\times\, h_\beta(x^{(i)})\left(1 - h_\beta(x^{(i)})\right)$$

$$\times\, w_1 \left.\frac{\partial h_1^{(i)}}{\partial w_{11}}\right|_{w_{11}=w_{11}^{\{k\}}}$$

$$h_1 = \sigma(w_{11}x_1 + w_{21}x_2 + b_1)$$

$$\frac{\partial h_1}{\partial w_{11}} = h_1(1-h_1)\,x_1$$

$$\boxed{\left.\frac{\partial h_1^{(i)}}{\partial w_{11}}\right|_{w_{11}=w_{11}^{\{k\}}} = h_1^{(i)}\left(1 - h_1^{(i)}\right)\left(x_1^{(i)}\right)}$$